Best Adaptive Tiling in a Rate-Distortion Sense

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The quad-tree data structure is applied widely in digital image processing and computer graphics for modeling spatial segmentation of images. A quad-tree is the tree in which each node (parent) has four descendants (children). In the present work we employ a quad-tree segmentation following by a coding algorithm on each image block, in rate distortion optimal sense, for a class of black and white images with smooth separation. We are trying to represent the boundary as a concatenation of linear segments which are the best approximation in the sense of least square error.

For compression application, coefficient location or side information has to be taken into account. We employ an operational rate-distortion sense that is similar to the approach used in [4] in finding best wavelet packet bases. A decision strategy based on optimizing rate distortion performance for each image block is designed so that the coder can decide if a block is worth to be further divided and coded with some appropriate quantization level.

The R-D measure is the most sensible for compression applications. The R-D function gives the trade-off between the rate (R) or the number of bits required to represent the signal, and the distortion (D) representing the squared error between the original signal and the approximation using R bits. An efficient way for optimizing rate-distortion is by merging through the minimization of the Lagrangian cost function defined as

$$J(\lambda) = D + \lambda R \tag{1}$$

The complete algorithm consists of the following parts: 1) Quad-tree decomposition algorithm is applied on the image up to some predefined level. 2) Each parent and its children (partition) are approximated by a linear segment in the least mean square error sense. 3) Pruning algorithm decides to prune or keep the children on the basic of rate distortion performance. To code a block with a separation segment we quantize its boundary using a certain quantization step depending on a given rate and assign the best quantized values to the end points of the separation line. This algorithm will provide us an approximation of the separation curve as a concatenation of linear segments as shown in Fig. 1. It is interesting to note that the underlying dictionary of "tiles" in the above algorithm is precisely the wedgelet dictionary [3].

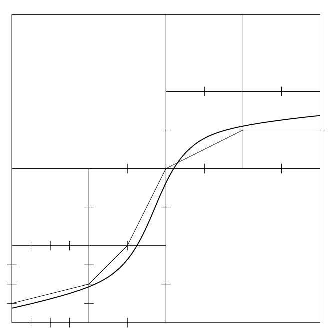

Figure 1: Approximation of a smooth separation curve where each block provides a linear segment.

For general piecewise smooth images, applying ridgelet transform [1, 2] on each of these image block separately will result in very few transform coefficients as ridgelet transform is well adapted for line-like singularities phenomenon. That means representation of smooth curve discontinuities as concatenation of linear segments via an adaptive quad-tree algorithm in conjunction with ridgelet transform will lead to a good compression scheme. We have investigated this scheme on simple black and white images with polynomial partitioning and results are promising and fulfilling the expectations of higher compression rates.

References

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